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CALCULATION OF THE SWEEP SPATIAL FABRIC SHELL TAKING INTO ACCOUNT THE FORMING PROPERTIES OF THE MATERIAL

Abstract: The article is devoted to the design of textile spatial shells. The purpose of this work is to develop a method for designing parts of a woven shell of a spatial form, taking into account the molding properties of the material. As a result of theoretical studies, an analytical method has been developed for obtaining a sweep of a tissue membrane around a dome-shaped surface.

Keywords: The cloth; textile shell; Shell design.

One of the methods for designing and manufacturing fabric volumetric casings is a method that takes into account the most important technological property of the mesh structure of fabric - the ability to take a volumetric shape by changing the angles between the warp and weft threads.

The quality of fabric products is largely determined by the validity of the developed design. One of the directions for intensifying and improving the quality of designing such products is the automation of the design processes of product parts. At the same time, the mathematical apparatus incorporated in the programs for calculating the sweep of the shells should provide high accuracy in determining the contours of the sweeps and a fairly accurate determination of the deformations of the tissue that occur during the formation of bulk surfaces.

The purpose of this work is to develop an analytical method for designing parts of a dome-shaped woven shell (a combination of areas of a sphere and a cone), taking into account the forming properties of the material.

To obtain a sweep above the specified shell, it is sufficient to consider the sweep of the $1 / 4$ part when the zero weft thread $U_{0}$ and the zero warp thread $\mathrm{S}_{0}$ are perpendicular to each other (Fig. 1). On the surface, the fabric mesh forms the curvilinear coordinate system $\mathrm{SM}_{00} \mathrm{U}$. The web of the fabric forms cells with side dimensions -lo along the warp threads and -ly along the weft threads. The calculation of the sweep parameters can be significantly accelerated by increasing the size of the tissue cells, but an increase in the dimensions of the sides leads to unacceptable errors in the calculation of parameters [1.2].

The lengths of the warp and weft threads in the cell were determined by linear interpolation $[3,4,5]$. When the ratio $\mathrm{S} / \mathrm{R} \leq 0.4$, the error does not exceed 0.02 mm , which ensures the required sweep accuracy.


Fig. 1. Scheme of a tissue shell of a spatial form
Based on this, to determine the unfolding of the shell, a sufficient accuracy can be obtained by dividing the fabric mesh into cells with side dimensions $1 \leq 40$ mmю Since the zero threads of the warp $\mathrm{S}_{0}$ and weft $\mathrm{U}_{0}$ lie on the perpendicularly directed meridians of the sphere, the coordinates of the nodal points of these threads Mio and Moj (Fig. 2) can be determined from the dependence

$$
\begin{equation*}
\mathrm{X}_{\mathrm{io}}=\mathrm{R}_{\mathrm{b}} \sin \alpha ; \mathrm{X}_{\mathrm{oj}}=0 ; \mathrm{Y}_{\mathrm{io}}=0 ; \mathrm{Y}_{\mathrm{oj}}=\mathrm{R}_{\mathrm{b}} \sin \alpha ; \mathrm{Z}_{\mathrm{io}}=\mathrm{R}_{\mathrm{b}} \cos \alpha ; \mathrm{Z}_{\mathrm{oj}}=\mathrm{R}_{\mathrm{b}} \cos \alpha \tag{1}
\end{equation*}
$$

where $\mathrm{i}=\mathrm{j}=0 ; 1 ; 2 ; \ldots . . ; \mathrm{k}$


Fig. 2. Zero warp and weft threads on the shell surface
$X_{i 0} ; Y_{i 0} ; Z_{i 0}-$ coordinates of points Mio, lying on the intersection of zero warp thread $\mathrm{S}_{\mathrm{O}}$ and j -th weft thread $\mathrm{Uj}, \mathrm{mm}$; Xoj; Yoj; Zoj - coordinates of points Moi, lying at the intersection of the zero thread of the weft $U_{o}$ and the $i-t_{h}$ warp thread $\mathrm{Si}, \mathrm{mm}$; lo and ly - sizes of fabric cells, respectively, along the warp and weft threads, mm ; Rb-radius of the sphere, mm ; Rк-radius of the lower part of the cone,
mm ; ho, hi, H- respectively the height of the tapered part and the entire product; $\alpha$ - is the radian measure of the arc, rad (Fig. 2).

To determine the coordinates of the nodal points on the zero warp and weft threads on the conical part of the surface, consider a unit vector that forms the cone. The coordinates of this vector will look like:

$$
\mathrm{a}=\frac{\mathrm{h}_{\mathrm{o}}}{\mathrm{R}_{\mathrm{b}}} \quad \mathrm{~b}=\frac{\sqrt{\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{h}_{\mathrm{o}}^{2}}}{\mathrm{R}_{\mathrm{b}}}
$$

Then the coordinates of the nodal points on the surface will be determined from the dependencies:

$$
\begin{align*}
& X_{\mathrm{io}}=\sqrt{\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{h}_{\mathrm{o}}^{2}}+\mathrm{al}(\mathrm{i}-\mathrm{k}) ; \quad \mathrm{Y}_{\mathrm{io}}=0 ; Z_{\mathrm{io}}=\mathrm{h}_{\mathrm{o}}+\mathrm{bl}(\mathrm{i}-\mathrm{k}) \\
& X o j=0 ; Y o j=\sqrt{\mathrm{R}_{\mathrm{b}}-\mathrm{h}_{\mathrm{o}}^{2}}+\mathrm{al}(\mathrm{j}-\mathrm{k}) ; Z \mathrm{oj}=\mathrm{h}_{\mathrm{o}}+\mathrm{bl}(\mathrm{j}-\mathrm{k}) \tag{2}
\end{align*}
$$

where $\quad i=j=k ; k+1 ; \ldots, N$
N - is the number of points falling on the tapered part
Determination of the number of points falling on the spherical and conical part of the surface is carried out according to the formulas:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{b}} \cos \mathrm{k} \alpha=\mathrm{h}_{\mathrm{o}} \quad \Rightarrow \quad \mathrm{k}=\frac{\left(\operatorname{arc} \mathrm{R}_{\mathrm{k}}^{2} \cos \frac{\mathrm{~h}_{\mathrm{o}}}{\mathrm{R}_{\mathrm{b}}}\right)}{\alpha} \quad \mathrm{N}=\frac{\left|M_{N} M_{K}\right|}{1} \tag{3}
\end{equation*}
$$

Next, it is necessary to determine the coordinate of the point -ho along the $Z$ axis, which determines the level of transition of the sphere into a cone.
According to $h_{1}=H-R_{b}$ coordinates $M_{N}$ и $M_{K}$ can be depicted as follows:

$$
\mathrm{M}_{\mathrm{N}}\left(-\mathrm{R}_{\mathrm{k}} ; \mathrm{h}_{1}\right) ; \mathrm{M}_{\mathrm{K}}\left(\sqrt{\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{h}_{\mathrm{o}}^{2}} ; \mathrm{h}_{\mathrm{o}}\right)
$$

Based on the fact that $\overrightarrow{M_{N} M_{k}} \perp \overrightarrow{O M_{k}} \quad$ we write

$$
\frac{\sqrt{\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{h}_{\mathrm{o}}^{2}}}{\mathrm{~h}_{\mathrm{o}}-\mathrm{h}_{1}}=\frac{\mathrm{h}_{\mathrm{o}}}{\sqrt{\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{h}_{\mathrm{o}}^{2}}}
$$

this implies $\mathrm{R}_{\mathrm{b}}{ }^{2}+\mathrm{h}_{\mathrm{o}}{ }^{2}+\mathrm{R}_{\mathrm{k}} \sqrt{\mathrm{R}_{\mathrm{b}}{ }^{2}-\mathrm{h}_{\mathrm{o}}{ }^{2}}=\mathrm{h}_{\mathrm{o}}{ }^{2}-\mathrm{h}_{1} \mathrm{~h}_{\mathrm{o}}$

$$
\begin{gathered}
\mathrm{R}_{\mathrm{k}}^{2} \mathrm{R}_{\mathrm{b}}^{2}-\mathrm{R}_{\mathrm{k}}^{2} \mathrm{~h}_{\mathrm{o}}^{2}=\mathrm{R}_{\mathrm{k}}^{4}-2 \mathrm{R}_{\mathrm{b}}^{2} \mathrm{~h}_{1} \mathrm{~h}_{\mathrm{o}}+\mathrm{h}_{1} \mathrm{~h}_{\mathrm{o}} \\
\mathrm{~h}_{\mathrm{o}}^{2}\left(\mathrm{~h}_{\mathrm{i}}^{2}+\mathrm{R}_{\mathrm{k}}^{2}\right)-2 \mathrm{R}_{\mathrm{b}}^{2} \mathrm{~h}_{1} \mathrm{~h}_{\mathrm{o}}+\mathrm{R}_{\mathrm{b}}^{2}\left(\mathrm{R}_{\mathrm{b}}^{2}-\mathrm{R}_{\mathrm{k}}^{2}\right)=0
\end{gathered}
$$

Solving the square equation, we find
$\mathrm{h}_{\mathrm{o}}=\frac{\mathrm{R}_{\mathrm{b}}{ }^{2} \mathrm{~h}_{1}+\sqrt{\mathrm{R}_{\mathrm{b}}{ }^{4} \mathrm{~h}_{1}{ }^{2}+\left(\mathrm{h}_{1}{ }^{2}+\mathrm{R}_{\mathrm{k}}{ }^{2}\right)\left(\mathrm{R}_{\mathrm{K}}{ }^{2}-\mathrm{R}_{\mathrm{b}}{ }^{2}\right) \mathrm{R}_{\mathrm{b}}{ }^{2}}}{\mathrm{~h}_{1}{ }^{2}+\mathrm{R}_{\mathrm{k}}{ }^{2}}=\frac{\mathrm{R}_{\mathrm{b}}{ }^{2} \mathrm{~h}_{1}+\mathrm{R}_{\mathrm{b}} \sqrt{\mathrm{R}_{\mathrm{K}}{ }^{2} \mathrm{~h}_{1}{ }^{2}+\mathrm{R}_{\mathrm{K}}{ }^{4}-\mathrm{R}_{\mathrm{b}}{ }^{2} \mathrm{R}_{\mathrm{k}}{ }^{2}}}{\mathrm{~h}_{1}{ }^{2}+\mathrm{R}_{\mathrm{k}}{ }^{2}}$

The next step is to calculate the coordinates Mij of the point. To calculate these coordinates, consider a cell formed by the intersection of warp and weft threads on a spherical surface (Fig. 3a).

Using vector properties [6;7], knowing the coordinates of the anchor points $\mathrm{M}_{00} ; \mathrm{M}_{01} ; \mathrm{M}_{10}$.

$$
\overrightarrow{r_{00}}=\left\{\mathrm{X}_{00} ; \mathrm{Y}_{00} ; \mathrm{Z}_{00}\right\} \quad \overrightarrow{r_{11}}=\left\{\mathrm{X}_{10} ; \mathrm{Y}_{10} ; \mathrm{Z}_{10}\right\} \quad \overrightarrow{r_{01}}=\left\{\mathrm{X}_{01} ; \mathrm{Y}_{01} ; \mathrm{Z}_{01}\right\}
$$

find the coordinates of the point $\mathrm{M}_{11}$ :
$\overrightarrow{r_{11}}=2 \frac{\overrightarrow{r_{01}}+\overrightarrow{r_{10}}}{\left(\overrightarrow{r_{01}}+\overrightarrow{r_{10}}\right)^{2}}\left(\overrightarrow{r_{00}}\left(\overrightarrow{r_{01}}+\overrightarrow{r_{10}}\right)\right)-\overrightarrow{r_{00}}=\frac{2 \cos \alpha}{1-\cos _{10} \mathrm{r}_{01}}\left(\overrightarrow{r_{01}}+\overrightarrow{r_{10}}\right)-\overrightarrow{r_{00}}$
Consider the case when the points go to the tapered part of the surface, since the cone is isometric to the plane, we write down the equation that describes the surface of the cone in a cylindrical coordinate system (Fig. 3b).

a

$\sigma$

Fig. 3. Cell of fabric on the surface: a - spheres; b - cone depicted in cylindrical coordinates

$$
\begin{equation*}
X_{i}=r_{i} \cos \beta_{i} ; \quad Y_{i}=r_{i} \sin \beta_{i} ; Z_{i}=c-\gamma r_{i} \tag{6}
\end{equation*}
$$

$\beta \mathrm{i}$ - rotation of the i -th vector projections from the OX axis;

$$
\mathrm{C}=\frac{1}{Z \mathrm{o}} ; \gamma=\frac{\mathrm{C}-z_{\mathrm{o}}}{\sqrt{\mathrm{R}_{\mathrm{W}}{ }^{2}-\mathrm{Z}_{0}{ }^{2}}}
$$

The corresponding equation of the unfolding of the cone in the plane with coordinates U ; V will be

$$
\begin{equation*}
\mathrm{U}_{1}=\gamma_{1} \mathrm{r}_{\mathrm{i}} \cos \beta_{\mathrm{i}} / \gamma_{1} \mathrm{~V}_{1}=\gamma_{1} \mathrm{r}_{\mathrm{i}} \sin \beta_{\mathrm{i}} / \gamma_{1} \quad \text { Where } \gamma_{1}=\sqrt{1+\gamma^{2}} \tag{7}
\end{equation*}
$$

We translate in the plane according to the above formulas $(6 ; 7)$ arbitrary points 1 , 2, 3 that are on the cone (Fig. 3 b) be:

$$
\mathrm{r}_{\mathrm{i}}=\sqrt{\mathrm{X}_{\mathrm{i}}^{2}+\mathrm{Yi}^{2}} \quad \beta_{\mathrm{i}}=\operatorname{arktg} \frac{\mathrm{Yi}}{\mathrm{X}_{\mathrm{i}}}
$$

The coordinates of the corresponding points on the plane will be

$$
\mathrm{U}_{\mathrm{i}}=\frac{\gamma_{\mathrm{i}} \mathrm{r}_{\mathrm{i}} \cos \beta_{\mathrm{i}}}{\gamma_{1}} \quad \mathrm{~V}_{\mathrm{i}}=\gamma_{\mathrm{i}} \mathrm{r}_{\mathrm{i}} \sin \beta_{\mathrm{i}} / \gamma_{1} . \overrightarrow{r_{4}} \text { Figure 1-it follows } \quad \vec{r}_{2}+\vec{r}_{3}-\vec{r}_{1}
$$

$$
\begin{array}{r}
\mathrm{r}_{4}=\frac{\sqrt{\mathrm{U}_{4}{ }^{2}+\mathrm{V}_{4}{ }^{2}}}{\gamma_{1}} \quad \beta_{4}=\frac{\operatorname{arktg} \frac{\mathrm{V}_{4}}{\mathrm{U}_{4}}}{\gamma_{1}} \\
\mathrm{X}_{4}=\mathrm{r}_{4} \cos \beta_{4} ; \quad \mathrm{Y}_{4}=\mathrm{r}_{4} \sin 4 ; \mathrm{Z}_{4}=c-\gamma \mathrm{r}_{4}
\end{array}
$$

Thus, the coordinates of the knot points of the fabric can be determined using the formulas presented, sequentially changing the numbers of the points $(i=j=1$ $\div \mathrm{N})$ to the edge of the product, since the coordinates of the initial points will always be known.

Thus, as a result of theoretical studies, an analytical method has been developed for obtaining a scan of a tissue shell of a spatial shape.

In the case of insufficient forming ability of materials, it is required to analytically determine the geometric dimensions and location of the cutouts, which allow obtaining a volumetric shape, taking into account the forming properties of the material without deforming the sides of the cells.

## References

1. Гусев Е.А. Машинно-ориентированный метод получения разверток проєктируемых деталей одежды./Автоматизированные системы управления технологическими процессами в легкой промышленности. Научные труды МТИЛП, 1985, -с 60-66.
2. Лопрандин И.В. Расчет оболочек и разветок одежды промыщленного производства.-М.: Легкая и пищевая промышленность,1982.-169 с.
3. Умнов А.Е.Аналитическая геометрия и линейная алгебра.-М.:МФТИ, 2011.-312с.
4. Воеводин В.В. Вычислительные основы линейной алгебра.-М.:Наука 1977.-303 с.
5. Якунин В.И. Геометрические основы сисмтем автоматизированного проектирования технических поверхностей.-М.:МАИ,1980.-86 с.
